April 19, 1888.

Admiral Sir G. H. RICHARDS, K.C.B., Vice-President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The Right Hon. Lord Sudeley was admitted into the Society.

The following Papers were read:—

I. "The Radio-Micrometer." By C. V. BOYS, A.R.S.M. Communicated by Professor A. W. RÜCKER, F.R.S. Received March 8, 1888.

(Abstract.)

In the full paper I have treated the subject of the radio-micrometer in such a manner as to arrive at the best proportions of the instrument. But I have first referred to the fact that the invention of an instrument of the kind was originally made by M. d'Arsonval, and it was in ignorance of this that I sent in my preliminary note.

The instrument consists essentially of a thermo-electric circuit suspended by a torsion fibre in a strong magnetic field. At first I have shown that the parts cannot be too thin nor the circuit too small until the limits imposed by practical considerations make further reduction objectionable. I have made the circuit of a bar of antimony and bismuth, with the ends joined by a hoop of copper wire.

I have at first taken the bar as an invariable, and shown how the copper wire may be proportioned to it to give the best results.

By "best" may be meant that which will give the greatest deflection, either for the weight or for the moment of inertia of the suspended parts.

Calling

W the weight of the bar and mirror (invariable),
w the weight of the copper wire (variable),
C the resistance of the bar (invariable),
r the resistance of the copper wire (variable),
l the length of the rectangle of copper, supposed 1 cm. wide,
u' the weight of a piece of copper of unit dimensions,
v the resistance of a piece of copper of unit dimensions,
a the sectional area of the copper wire,

I have shown that—

$$\text{The best sectional area, } a = \sqrt{\left(\frac{Wv}{u'C}\right)},$$

$$\text{Twice the best length, } 2l = 1 + \sqrt{\left(\frac{CW}{u'v}\right)},$$

and that the best number of turns of wire is 1.

The numerical values for a particular bar $10 \times 5 \times \frac{1}{4}$ mm. are—

$$a = 0{\cdot}002007 \text{ sq. cm.}, \qquad l = 4{\cdot}621 \text{ cm.}$$

If the breadth be also a variable, the best rectangle is a square of infinite size made of the same wire, which is always the best, whatever shape, size, or number of turns the circuit may have.

The best circuit with respect to moment of inertia is that which is practically required, because a convenient period of oscillation must be made use of, and so the torsion must be supposed to vary as the moment of inertia. A difficulty was found in working the expression for this, which was entirely overcome by supposing the wire where it crosses the axis to have a sectional area proportional to its distance from the axis, except in its immediate neighbourhood. On this supposition the resistance and the moment of inertia of the upper side of the rectangle are each equal to that of half the same length of copper wire on the sides, and thus not only has the best variation been found, but, what is more important, the coefficients for resistance and moment of inertia have been made identical, which is required in order to put the equations into a simple form.

The expressions found with respect to weight are now applicable to moment of inertia if certain changes are made. Thus, the figure 1 in the expression for length must be replaced by $\frac{1}{2}$. The moment of inertia of the active bar K must replace its weight W, and the moment of inertia of a unit piece of copper at 5 mm. from the axis u must replace its weight u'.

It is thus found that the expressions for

$$\text{The best sectional area, } a = \sqrt{\left(\frac{Kv}{uC}\right)}.$$

And this is true whatever length or number of turns the circuit may have.

$$\text{Twice the best length, } 2l = \tfrac{1}{2} + \sqrt{\left(\frac{KC}{uv}\right)}.$$

As before, the best number of turns is 1.

The numerical values are—

$$a = 0{\cdot}00102 \text{ sq. cm.}, \qquad l = 2{\cdot}337 \text{ cm.}$$

These expressions give the proportions which will produce the greatest deflection. But in case of a strong magnet the resistance to the motion is so great as to be more than sufficient to make the movement dead beat, and this is inconvenient. I have therefore introduced the effect of this resistance into the equations, and found expressions for the best circuit that is just dead beat.

Calling H' the least magnetic field that will make the circuit dead beat,

G the conductivity of the whole circuit,

K' the moment of inertia of the whole circuit,

I have shown that—

$$H' = 2\sqrt{\frac{\pi}{\tau}} \cdot \frac{\sqrt[4]{K'}}{l\sqrt{G}},$$

and that the greatest sensibility of a circuit that is just dead beat is—

$$S = 2\sqrt{\frac{\pi}{\tau}} \cdot \frac{\sqrt{G}}{K'^{\frac{3}{4}}}.$$

From these it is found that the best sectional area is reduced to about three-fourths its previous value, but that the shorter the rectangle of copper the better, until the greatest magnetic field that can be made use of practically is reached.

On considering variations of breadth in the circuit, it is found that if the upper side of the rectangle—that which crosses the axis—is neglected, the sensibility is independent of the breadth, and that the following relations hold:—

$$\text{Best } a = \frac{1}{b}\sqrt{\frac{Kv}{uC}},$$

$$\text{Best } l = \frac{1}{2b}\sqrt{\frac{KC}{uv}},$$

when b is the breadth, and that what I have called the greatest efficacy E_k, *i.e.*, sensibility in a given field, is—

$$E_k = \frac{1}{8\sqrt{(KCuv)}}.$$

Since the cross wire becomes increasingly mischievous with an increasing breadth of circuit, b cannot be made too small.

Further, it appears that the copper wire should have the same moment of inertia and resistance as the invariable parts of the circuit.

Other expressions are given, but it may be sufficient to state here

that the circuit which is best according to the rules given by these equations is seven times as good as the best previously found.

I have then shown that the mirror must be of such a size as to have a moment of inertia one-third of that of the active bar. In the particular case considered, where the active bar consists of two pieces, one antimony and one bismuth, $5 \times 1 \times \frac{1}{4}$ mm., at a mean distance of 1 mm. apart, the diameter of the mirror should be $2\frac{3}{4}$ mm. This size both theoretically should, and practically does, enable one with certainty to observe a deflection of $\frac{1}{4}$ mm. on a scale 1 metre distant.

General considerations show that the antimony-bismuth bars cannot have too small a sectional area, but that the length when already short is only involved in a secondary manner.

It is shown that the heat in the circuit is equalised mainly by conduction, which is thirty times as effective as the Peltier action.

It is found necessary to screen the antimony and bismuth from the magnetic field by letting them swing in a hole in a piece of soft iron buried in the brass work.

I have shown that the instrument imagined in the preliminary note would be so much more than dead beat that it would not be possible to use it advantageously, but on making a corresponding calculation for the best circuit, now found, using conditions which have been proved by practice to work well together, a difference of temperature of one ten-millionth of a degree centigrade is by no means beyond the power of observation.

The figures given by an actual comparison between the newest instrument and one of the original pattern is very favourable to the former.

In conclusion, I have explained the peculiar action of the rotating pile, and have shown that it is different from that figured in Noad's 'Electricity and Magnetism.'

II. "On Hamilton's Numbers. Part II." By J. J. SYLVESTER, D.C.L., F.R.S., Savilian Professor of Geometry in the University of Oxford, and JAMES HAMMOND, M.A., Cantab. Received March 9, 1888.

(Abstract.)

§ 4. *Continuation, to an infinite number of terms, of the Asymptotic Development for Hypothenusal Numbers.*

In the third section of this paper ('Phil. Trans.,' A., vol. 178, p. 311) it was stated, on what is now seen to be insufficient evidence, that the asymptotic development of $p-q$, the half of any hypothenusal